University of Saskatchewan

Department of Mathematics & Statistics

MATH 223.3 – FINAL EXAMINATION

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CLOSED BOOK, NO CALCULATORS, NO ELECTRONIC DEVICES

The exam is in three parts, Part A, Part B and Part C.
Use a different Exam Booklet for each part.
Please show all of your work. The marks for each question are shown by [x]

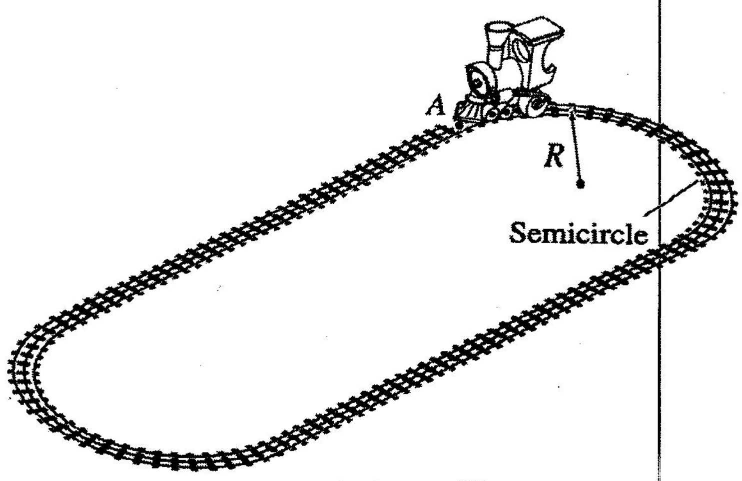
PART A. 34 MARKS.
Answer the following questions in an Exam Booklet and label it PART A.

[7] 1. Find the area of the parallelogram with vertices $(1, 2, 3), (4, 3, 7)$, check vert
$(-1, 3, 6)$, and $(2, 4, 10)$. Find the equation of the plane containing these points.

[4] 2. Find the distance between the line $x = 1 - t, y = 2 + 3t, z = 4 - 2t$
and the plane $2x + 4y + 5z = 10$. check not parallel

[7] 3. A particle has a trajectory defined by $x = t, y = t^2, z = t^2$, where t is time.
Find its *velocity, speed* and *acceleration* at any time. What are the *normal* and *tangential* components of its *velocity* and *acceleration*?

[6] 4. A toy train travels *at constant speed* around an oval track which consists of semi-circular segments joined together by straight segments as shown in the figure below.



This problem investigates why the train tends to wobble or even derail at the point A where the semicircular segment joins the straight line segment of rail.

(i) The position of the train at time t can be described by a vector function $\mathbf{r}$. Find $\mathbf{r}(t)$ which describes the position of the train before the point A and after the point A.

(ii) Show that, with your choice of $\mathbf{r}(t)$, the speed of the train is constant.

(iii) Show that the acceleration is discontinuous at A.

[5] 5. Find the directional derivative of $f(x, y, z) = x^3 y \sin z$ at $(3, -1, -2)$ along the line $x = 3 + t, y = -1 + 4t, z = -2 + 2t$ in the direction of *decreasing* x.

[5] 6. Find $\dfrac{dz}{du}$ if $z = x^2 y v^3$ and $x = u^3 + 2u,\ y = \ln(u^2 + 1)$, and $v = ue^u$.

PART B. 34 MARKS.

Answer the following questions in an Exam Booklet and label it PART B.

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[5] 7. Evaluate the double integral of $(4 - y^2)^{\frac{3}{2}}$ over the region bounded by the circle $x^2 + y^2 = 4$ in the first quadrant of the xy-coordinate plane.

[5] 8. Use a double integral to find the volume of the solid of revolution obtained by rotating the region bounded by the curves $y = x^2 - 2,\ y = 0$ about the line $y = -1$.

[5] 9. Use double integrals to find the centroid of the region bounded by the curves $y = e^x,\ y = 0,\ x = 0$, and $x = 1$.

[7] 10. Evaluate a double iterated integral to find the surface area of $x^2 + y^2 + z^2 = 2$ inside the cone $z = \sqrt{x^2 + y^2}$.

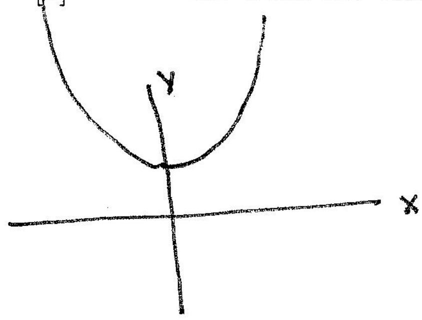
[5] 11. Evaluate the triple integral of $x + y + z$ over the region V which is bounded by $z = 1 - x^2 - y^2$ and $z = 0$.

Hint:

(i) First show that the integrals of x and y are zero.

(ii) Integrate z by converting to a convenient coordinate system.

[7] 12. Find the volume bounded by the surfaces $y + z = 2,\ z = 0$, and $y = x^2 + 1$.



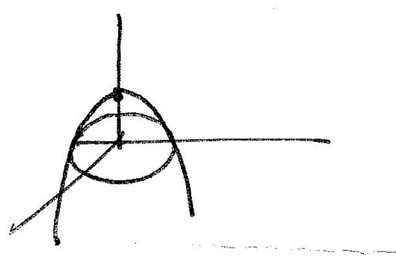
$z:0 :\quad x^2 + y^2 = 1$

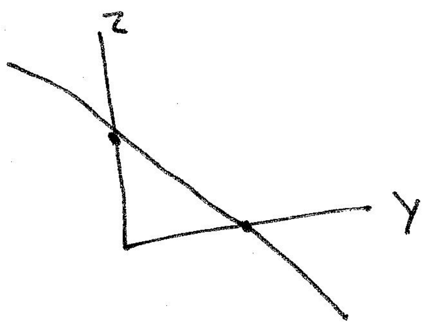
$z:1 :\quad x^2 + y^2 = 0$

~~$z:2 :\quad x^2 + y^2 = -2$~~

$z:-1 :\quad x^2 + y^2 = 2$

$z = -(x^2 + y^2 - 1)$





15
3
45
-10
+3
35
3
38/15

PART C. 32 MARKS.

Answer the following questions in an Exam Booklet and label it PART C.

[8] 13. Verify the identities

(a) $curl(grad\ f) = \nabla \times (\nabla f) = \mathbf{0}$

(b) $div(curl\ \mathbf{F}) = \nabla \cdot (\nabla \times \mathbf{F}) = 0$

where $f(x, y, z)$ is a sufficiently differentiable scalar field and
$\mathbf{F}(x, y, z) = P(x, y, z)\hat{\mathbf{i}} + Q(x, y, z)\hat{\mathbf{j}} + R(x, y, z)\hat{\mathbf{k}}$
is a sufficiently differentiable vector field

[5] 14. Evaluate the line integral $\displaystyle\int_c ydx + xdy + zdz$, where c is the curve

$z = x^2 + y^2,\ \ x + y = 1$ from $(1, 0, 1)$ to $(-1, 2, 5)$.

[5] 15. Show that the line integral $\displaystyle\int_c 3x^2y^3dx + 3x^3y^2dy$ is independent of path.

Evaluate it when c is the curve $y = e^x$ from $(0, 1)$ to $(1, e)$.

[5] 16. Use Green's theorem to evaluate the line integral

$\displaystyle\oint_c (xy^2 + 2x)dx + (x^2y + y + x^2)dy,$ $\quad y^2 = \sqrt{4+x^2}$

where c is the boundary of the region enclosed by $y^2 - x^2 = 4,\ x = 0$ and $x = 3$ and the direction of the circuit is counter clockwise.

[9] 17. Find the point(s) on the surface $z^2 = 1 + xy$ closest to the origin $(0, 0, 0)$.

Hints:

(a) Over what region in the xy-coordinate plane does the surface lie? In particular, what is the boundary of the region?

(b) It is simpler to use $(distance)^2$ since it is a minimum when $(distance)$ is a minimum.

$$\begin{vmatrix} \partial/\partial x & \partial/\partial y & \partial/\partial z \\ y & x & z \end{vmatrix}$$

$\cancel{z_y} = z_y - x_z \neq 0$

END